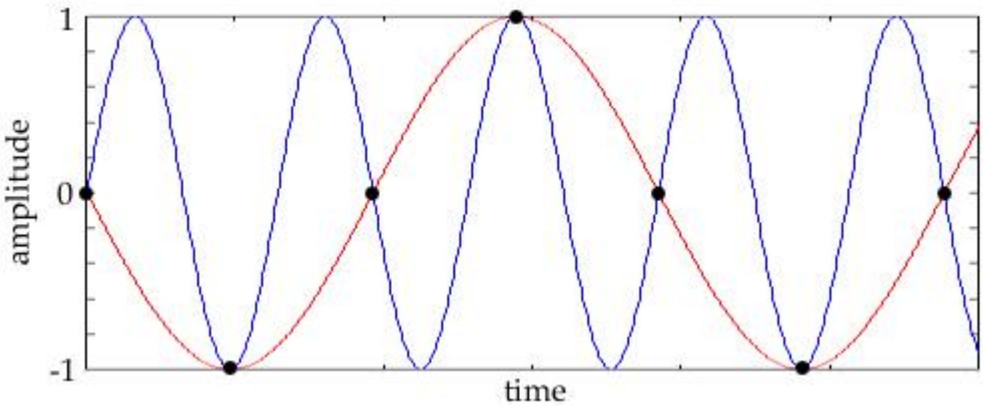
CHOIR NOTES - MARCH 2019

In this month's Choir Notes I want to talk about harmony. Harmony as in "the combination of simultaneously sounded musical notes to produce a pleasing effect". I do this with some trepidation because (i) I sing in the tenor section where harmony can often be hard to come by, and (ii) our organist Cynthia knows about a zillion times more about harmony than I do and will be correcting my homework if I get anything wrong!

While I have no formal musical training, I did study maths at school and spent an inordinate amount of time, it seemed, messing around with fractions. So what does this have to do with harmony? As it turns out, quite a lot – something which I find quite fascinating in a strange sort of way because the mathematics of harmony has had a big influence on how we write, listen to and play music.

It works something like this. If you play two notes on a piano which are an octave apart at the same time, the two notes sound almost identical. You may not even realise two notes are being played. This is because the string producing the higher note is vibrating at exactly twice the speed of the lower string (a ratio of 2:1, or as a fraction 2/1). On a graph it would look something like this.



The more wavy line (the higher note) and the less wavy line (the lower note) come together every two vibrations (or cycles). They "fit" like fingers in a glove. Your ear tells you that the two notes are "consonant" or "harmonious". If your piano goes out of tune, the two notes will no longer sound so good as they will not have that 2/1 ratio. The notes would be "dissonant" because they do not fit together any more.

This principle works for other fractions too. If the higher string vibrates at one and a half times the speed of the lower string (3/2 as a fraction) that also sounds "good" because the strings will come together every 3 cycles. That is not as often as 2 but still a lot! 4/3 works in the same way (every 4 cycles). As the fractions get smaller, the sound becomes less harmonious but they all sound reasonably good. Your ear is very good at identifying these natural resonances.

Most musical instruments are designed in a way that makes use of these natural harmonies. In fact, the notes in the twelve note scale we use in western music are all based on intervals that can be expressed in fractions. Take a look at the third column in the (scary-looking) table below ("ratio to unison using fractions"). It is really rather clever don't you think? Provided you ignore the occasional "weird" fraction, such as 11/6, there is a lot of harmony to be found by combining the bottom note ("unison") with other notes on this "fractional" scale.

Note	Interval	Ratio to Unison using Fractions	Ratio to Unison using Equal Temperament	Difference
1	Unison	1/1 = 1.0000	1.0000	0
2	Minor Second	12/11 = 1.0909	1.05946	+3.1%
3	Major Second	9/8 = 1.1250	1.12246	-0.3%
4	Minor Third	6/5 = 1.2000	1.18921	-1.1%
5	Major Third	5/4 = 1.2500	1.25992	+1.0%
6	Fourth	4/3 = 1.3333	1.33483	+1.5%
7	Diminished Fifth	7/5 = 1.4000	1.41421	+1.4%
8	Fifth	3/2 = 1.5000	1.49831	-0.2%
9	Minor Sixth	8/5 = 1.6000	1.58740	-1.3%
10	Major Sixth	5/3 = 1.6667	1.68179	+1.5%
11	Minor Seventh	7/4 = 1.7500	1.78180	+3.2%
12	Major Seventh	11/6 = 1.8333	1.88775	+5.6%
13	Octave	2/1 = 2.0000	2.0000	0

There is just one teeny weeny problem (and if you don't like maths please look away now). The ratio (or interval) between notes 1 and 5 on the fractional scale (which is 5/4 - yum) is not the same as the ratio between notes 3 and 7 on the same scale (which is 56/45 - yuk). This may not seem like a very big difference, but your ear will tell you otherwise. Tunes which sound nice in one key are going to sound fairly rubbish in other keys if you tune your instrument using the fractions in column 3.

Fortunately, we can fix this problem by making sure that the intervals between each note in the scale are the same. We do this by adjusting the tuning of each note in column 3 so that it matches the tuning in column 4. The notes in column 4 are all spaced exactly the same distance apart relative to each other (the ratio is the twelfth root of 2 in case you are interested). In some cases this adjustment is very small (the "fifth" is only 0.2% different) and in others the difference is quite big (the major seventh). These differences do make the listening experience a little less good (fractions do sound better than decimals), but the benefits of being able to play tunes in lots of different keys outweigh the slight reduction in listening pleasure caused by moving away from natural tuning.

At this point I can hear Cynthia tut tutting at the number of errors I have made already in attempting to explain this in a simple way, so this may be a good time to cut my losses and sign off. If you have made it this far, you are probably a mathematician, and well done! If you haven't, you are probably enjoying a nice cup of tea which is equally admirable. ☺

The moral of the tale is that musical harmonies are not perfect, but that is ok because neither are the tenors!!

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